MATH 306 Workshop

Important Theorems: (you should also review all the definitions)
Invertibility: 3.54, 3.56
Isomorphism: 3.59
Operator: 3.69

1. Write down the definitions:
    a. Invertible
    b. Isomorphism & Isomorphic
    c. Operator
2. What can we say about an operator which is injective?
3. A linear map T from $\mathcal{P}_{10}(\mathbb{F})$ to $\mathbb{F}^{3,3}$ has range which is isomorphic to $\mathbb{F}^{5}$. What is the dimension of null(T)?
4. Let B denote the basis 1, z, z^2 for $\mathcal{P}_2(\mathbb{F})$ and let B' denote the basis 1, $z - 1$, $z^2 - 1$. Find the change of basis matrix from B to B'. Use this matrix to find the representation of the polynomial $p(z) = 1 + 2z - 4z^2$ in terms of B'.

5. $Define\ a\ linear\ map\ T\ from\ \mathcal{M}_2(\mathbb{R})\ to\ \mathcal{P}_2(\mathbb{R})\ by$

$$T\left(\begin{pmatrix} a & b \\ c & d \end{pmatrix}\right) = c + d * z + (c - d) * z^2$$

$Find\ \mathcal{M}(T, B, B')\ where\ B = \left\{\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\right\}\ and\ B' = \{z^2, z, 1\}$

6. True or False (if false, give a counterexample):

   a. Given a linear map T. If the dimension of its domain is equal to the dimension of its codomain, then this map is an operator.

   b. Given a linear map T. If T is an operator, then it has to be one-to-one (injective) and onto (surjective).

   c. Given a linear map T. T is injective if and only if null T = {0}.